

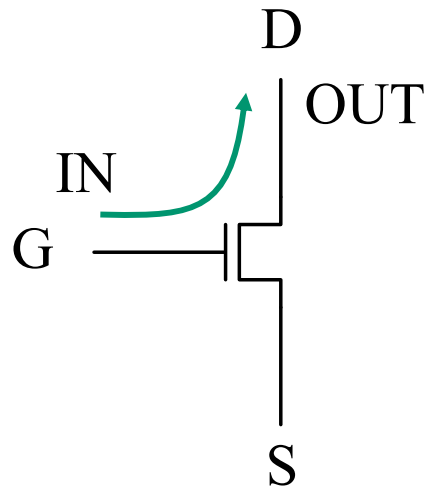
10. Basic amplifiers

Kanazawa University
Microelectronics Research Lab.
Akio Kitagawa

10.1 The foundations of Amplifiers

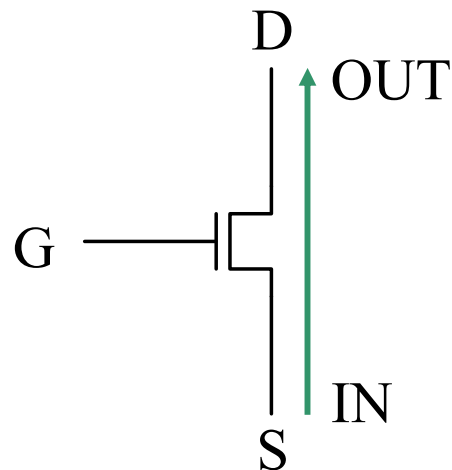
Signal transmission and common terminal

Common source



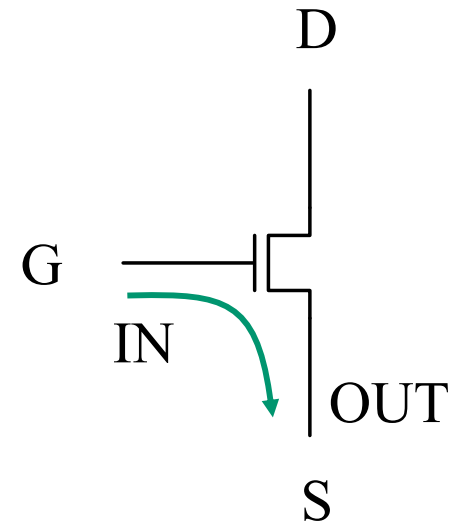
Inverting

Common gate



Non-inverting

Common drain



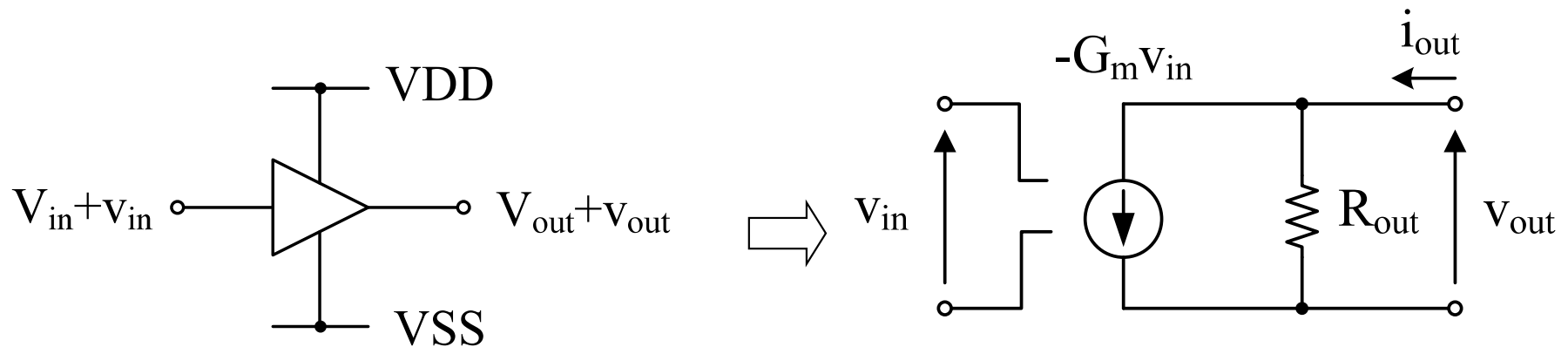
Non-inverting

Configurations of a basic amplifiers

Types of amplifiers

	Common source	Common gate	Common drain
Source	Common	Input	Output
Gate	Input	Common	Input
Drain	Output	Output	Common
Function	Voltage Amplifier H→L impedance	Voltage Amplifier L→H trans-impedance	DC level shift H→L impedance

Voltage gain of amplifiers



Small-signal equivalent circuit (Behavior model)

$$A_V \equiv \frac{v_{out}}{v_{in}} = \frac{-(-G_m \cdot v_{in}) \cdot R_{out}}{v_{in}} = G_m \cdot R_{out}$$

Assumption $\left\{ \begin{array}{l} \text{Impedance of input signal} = 0 \\ \text{Impedance of output load} = \infty \end{array} \right.$

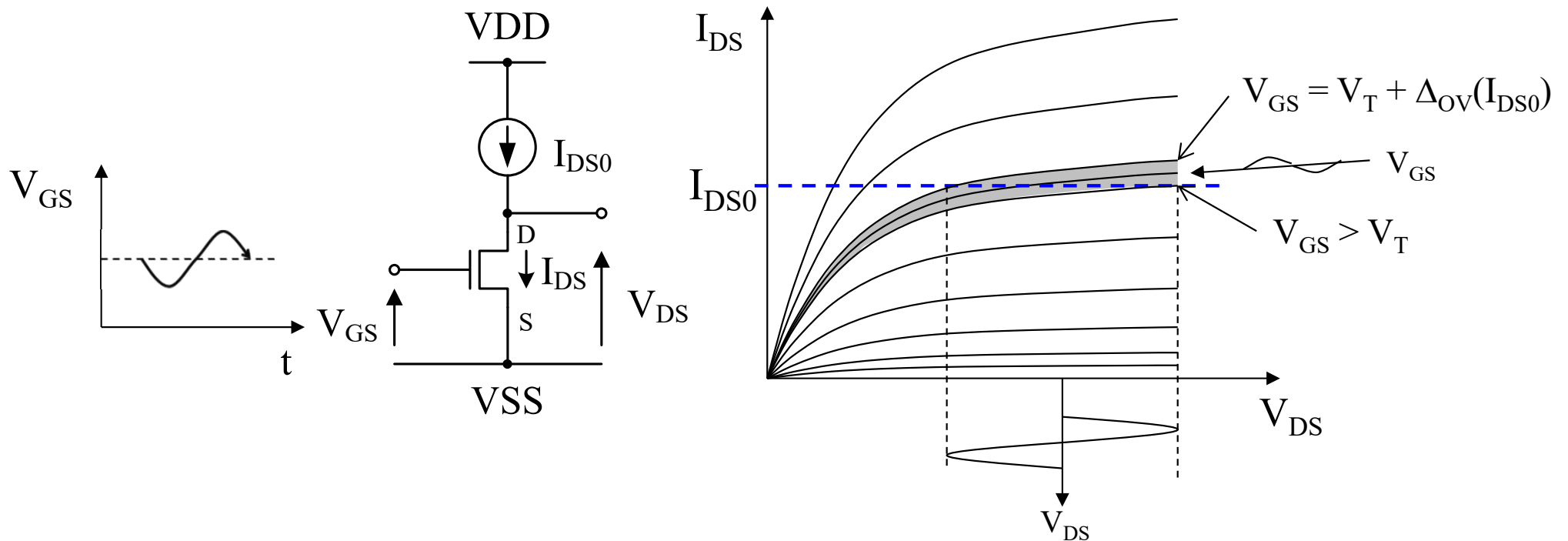
Definition of G_m , R_{out}

$$\left\{ \begin{array}{l} G_m = - \frac{i_{out}}{v_{in}} \Big|_{v_{out}=0} \\ R_{out} = \frac{v_{out}}{i_{out}} \Big|_{v_{in}=0} \end{array} \right.$$

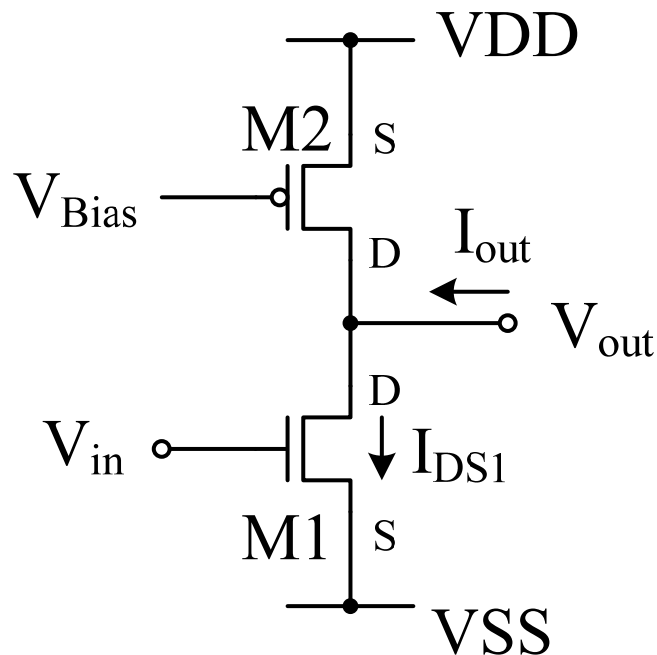
Common-source (CS) amplifier

Current source load I_{DS0} (MOSFET current source)

- Bias current I_{DS0} is supplied to M1
- Current source works as a large load resistance (Large voltage gain)



Voltage gain of CS amplifier



$$G_m \equiv -\frac{i_{out}}{v_{in}} \Big|_{v_{out}=0} = -\frac{i_{ds1}}{v_{gs1}} = -g_{m1}$$

$$R_{out} \equiv \frac{v_{out}}{i_{out}} \Big|_{v_{in}=0} = \frac{1}{g_{ds1} + g_{ds2}} = r_{ds1} // r_{ds2}$$

Please draw a small signal equivalent circuit for yourself.

$$A_v \equiv G_m \cdot R_{out} = \frac{-g_{m1}}{g_{ds1} + g_{ds2}} = -g_{m1} \cdot (r_{ds1} // r_{ds2}) = -g_{m1} \frac{r_{ds1} r_{ds2}}{r_{ds1} + r_{ds2}}$$

Setting of the bias current (1)

I-V characteristic in saturation region
($V_{DS} > V_{GS} - V_T$)

$$I_{DS} = \frac{1}{2} \frac{W}{L} \mu_n \cdot C_{OX} (V_{GS} - V_T)^2 \{1 + \lambda(V_{DS} - \Delta_{OV})\}$$
$$= \frac{\beta}{2} (V_{GS} - V_T)^2 \{1 + \lambda(V_{DS} - \Delta_{OV})\}$$

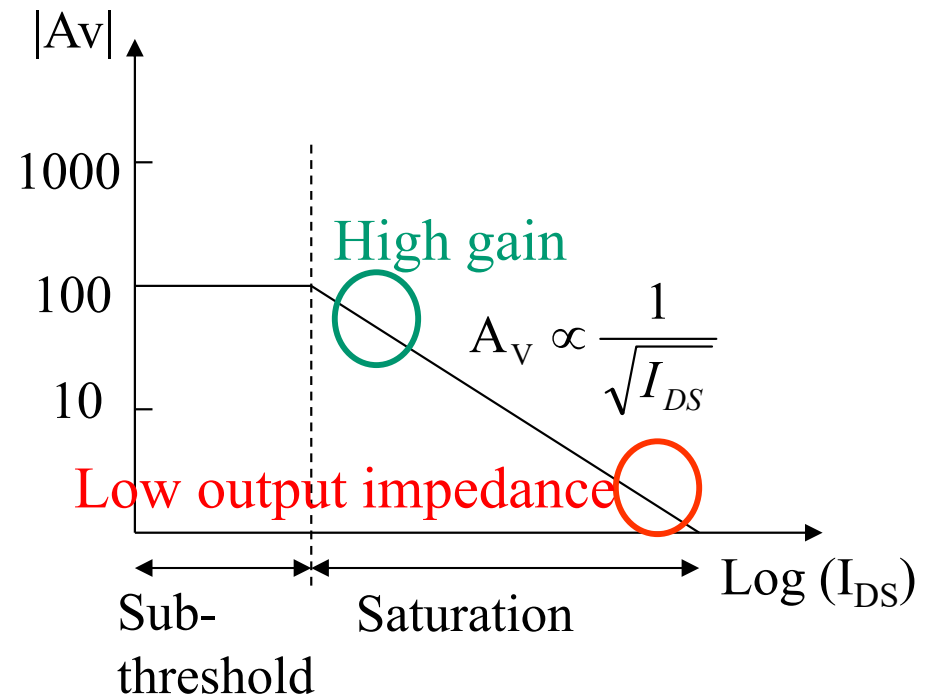


$$g_m \equiv \frac{\partial I_{DS}}{\partial V_{GS}} \approx \beta (V_{GS} - V_T) = \sqrt{2\beta \cdot I_{DS}}$$

$$g_{ds} \equiv \frac{\partial I_{DS}}{\partial V_{DS}} = \frac{\beta}{2} (V_{GS} - V_T)^2 \lambda \approx \lambda \cdot I_{DS}$$

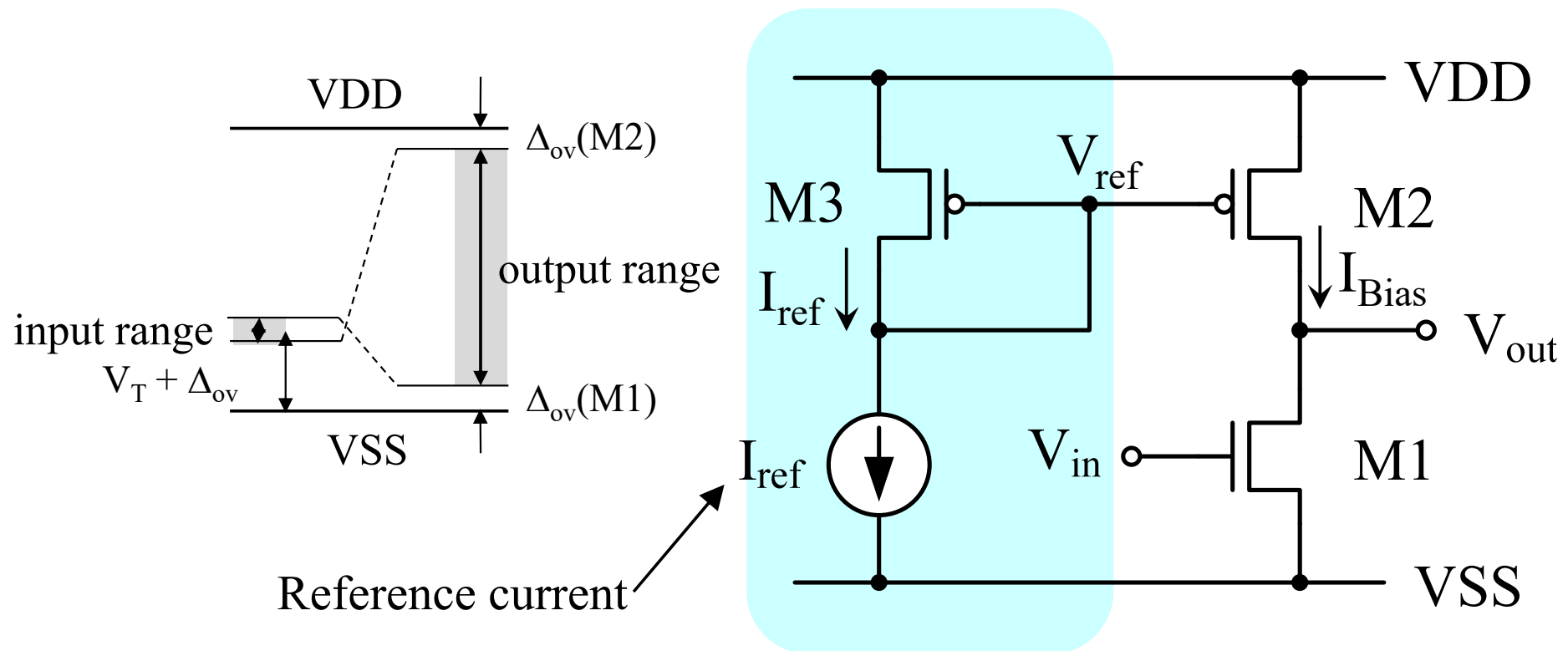
Setting of the bias current (2)

$$\begin{aligned} A_V &= -\frac{g_{m1}}{g_{ds1} + g_{ds2}} \\ &= -\frac{\sqrt{2\beta_1 I_{DS}}}{\lambda_1 I_{DS} + \lambda_2 I_{DS}} \\ &= -\frac{\sqrt{2\beta_1}}{\lambda_1 + \lambda_2} \frac{1}{\sqrt{I_{DS}}} \end{aligned}$$



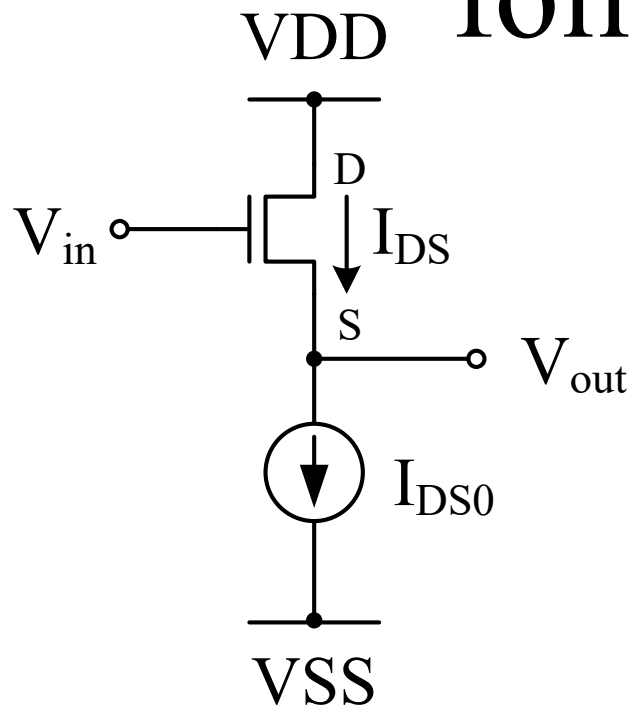
NOTE: g_m/I_{DS} is a reference index of the voltage gain in saturation region.

Bias circuit for CS amplifier

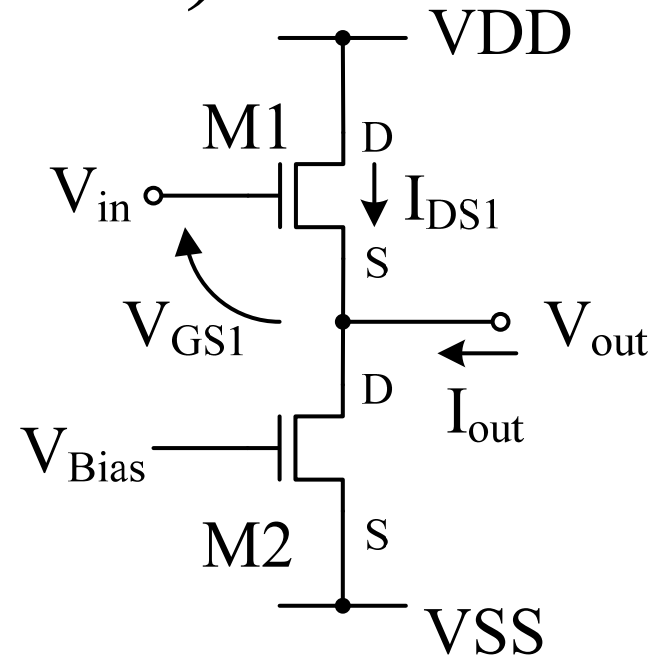


The bias current is supplied to M1 by the current mirror.

Common drain amplifier (Source follower or SF)



⇒
MOSFET
current source



$$\left\{ \begin{aligned} G_m &\equiv -\frac{i_{out}}{v_{in}} \Big|_{v_{out}=0} = -\frac{-i_{ds}}{v_{gs}} = g_{m1} \\ R_{out} &\equiv \frac{v_{out}}{i_{out}} \Big|_{v_{in}=0} = \frac{1}{g_{ds1} + g_{ds2} + g_{m1}} \end{aligned} \right.$$

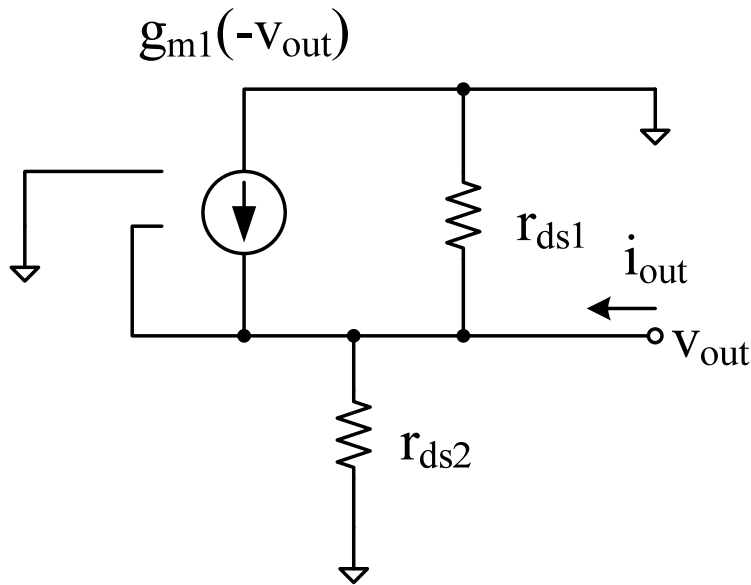
(See next slide)

$$A_V \equiv G_m \cdot R_{out} = \frac{1}{1 + \frac{1}{g_{m1} \cdot (r_{ds1} // r_{ds2})}} \approx 1$$

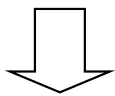
when $\frac{g_{m1}}{g_{ds1} + g_{ds2}} = g_{m1} \cdot (r_{ds1} // r_{ds2}) \gg 1.$

Output resistance of source follower

Small-signal equivalent circuit



Low output resistance



Small voltage gain and large current gain

$$V_{GS1} = V_{in} - V_{out}$$

$$V_{in} = \text{const.}$$

$$\therefore v_{gs1} = -v_{out}$$

$$i_{out} - g_{m1} \cdot v_{out} = \left(\frac{1}{r_{ds1}} + \frac{1}{r_{ds2}} \right) \cdot v_{out}$$

$$\begin{aligned} R_{out} &= \left(\frac{i_{out}}{V_{out}} \right)^{-1} \\ &= \frac{1}{g_{ds1} + g_{ds2} + g_{m1}} = \frac{1}{\frac{1}{(r_{ds1} // r_{ds2})} + g_{m1}} \\ &= \frac{r_{ds1} // r_{ds2}}{1 + g_{m1} \cdot (r_{ds1} // r_{ds2})} \approx \frac{1}{g_{m1}} \end{aligned}$$

DC level shifter

$$I_{DS0} = \frac{\beta}{2} (V_{in} - V_{out} - V_T)^2$$

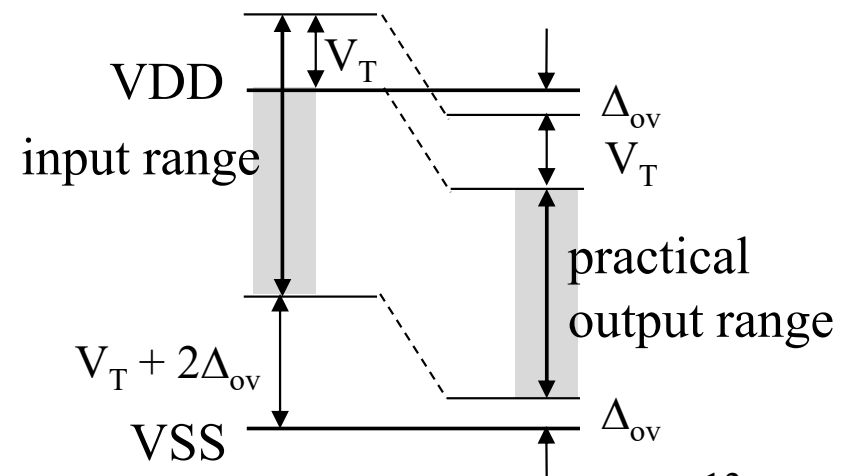
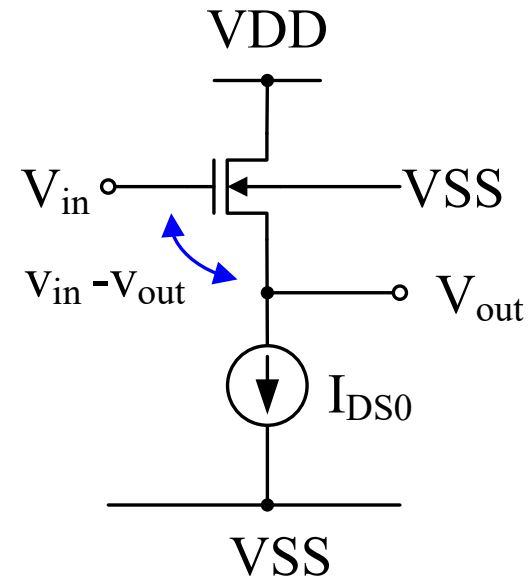
$$V_{out} = V_{in} - V_T - \sqrt{\frac{2I_{DS0}}{\beta_1}}$$

Level shift

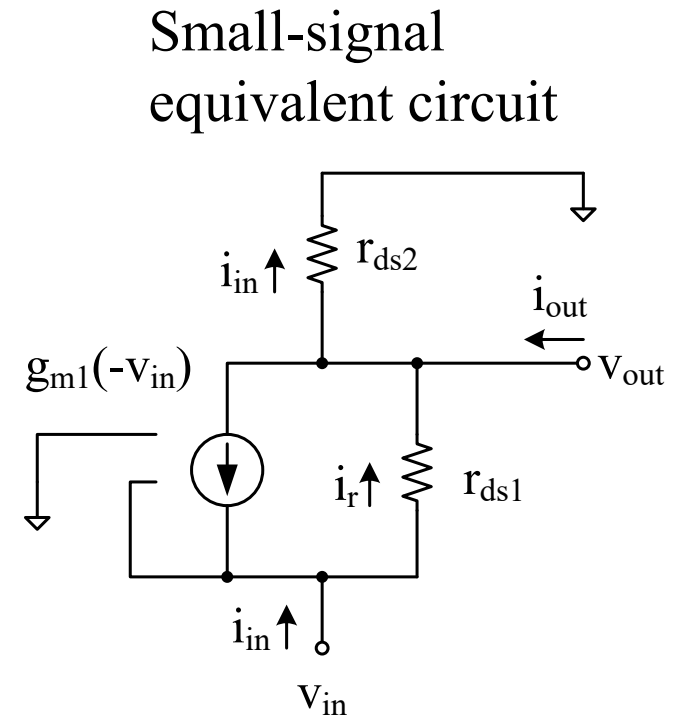
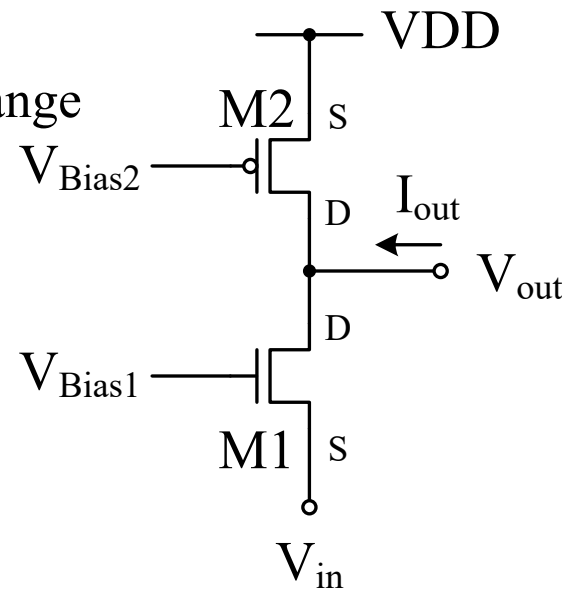
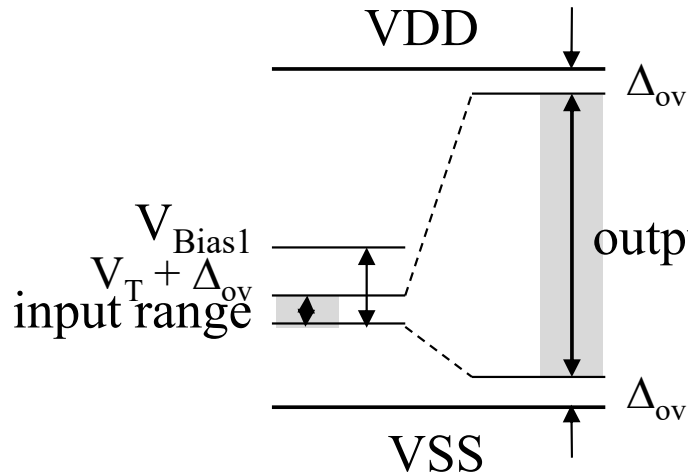
when $V_{in} = VDD$,

$$V_{out} = VDD - V_T - \sqrt{\frac{2I_{DS0}}{\beta_1}}$$

The voltage range of output is restricted in lower half of possible output range.



Common gate (CG) amplifier



$$V_{GS1} = V_{Bias1} - V_{in}$$

$$V_{Bias1} = \text{const.}$$

$$\therefore v_{gs1} = -v_{in}$$

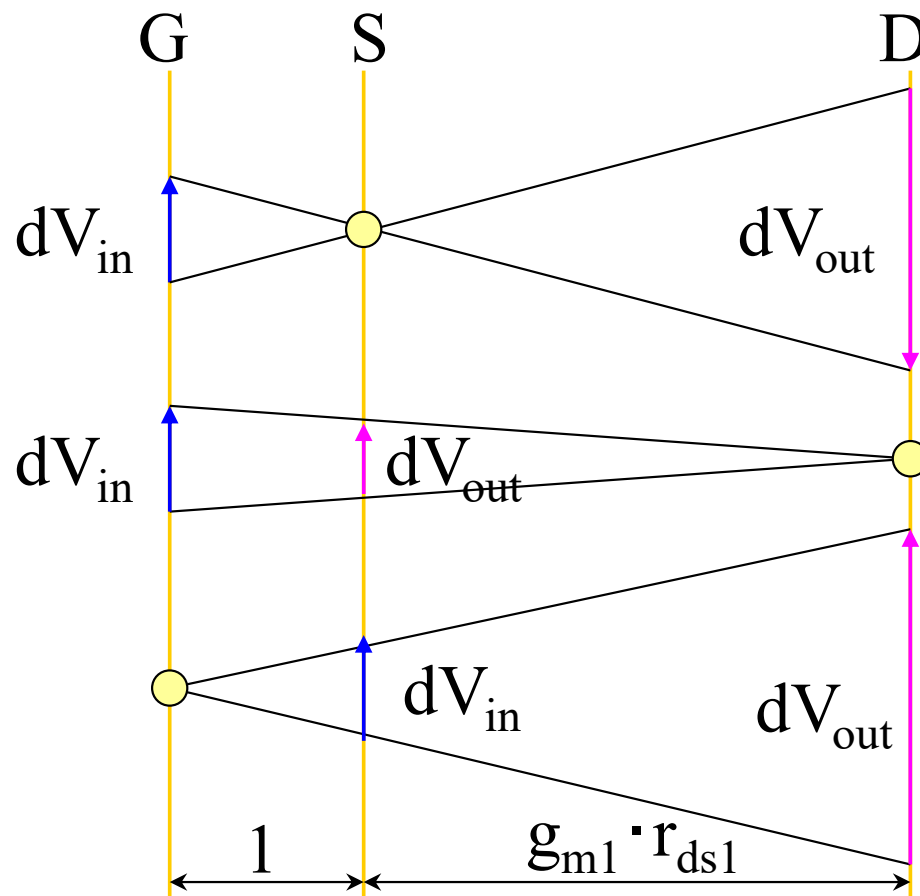
If $i_{out} = 0$, (with no external load)

$$\begin{cases} v_{out} = r_{ds2} i_{in} = r_{ds2} (g_{m1} \cdot v_{in} + i_r) \\ i_r = \frac{1}{r_{ds1}} (v_{in} - v_{out}) \end{cases}$$

$$A_V = \frac{r_{ds2}}{r_{ds1} + r_{ds2}} (1 + g_{m1} r_{ds1})$$

Summary of the basic amplifiers

The common terminal is regarded as the fulcrum of the lever.



If $r_{ds2} \doteq \infty$ (Ideal current load)

$$A_V = -g_{m1} r_{ds1} \quad (\text{Common source})$$

$$A_V = \frac{g_{m1} r_{ds1}}{1 + g_{m1} r_{ds1}} \quad (\text{Common drain})$$

$$A_V = 1 + g_{m1} r_{ds1} \quad (\text{Common gate})$$

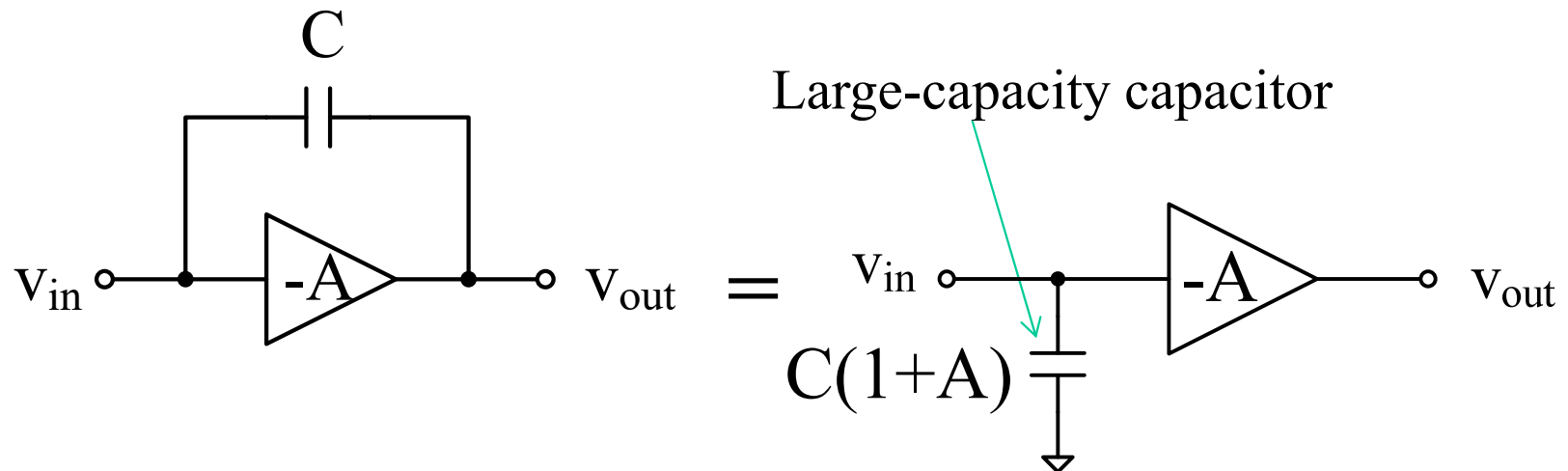
10.2 Cascode amplifier

Gain enhancement technique

- $A_v = G_m \cdot R_{out} = -\frac{\sqrt{2\beta_1}}{\lambda_1 + \lambda_2} \frac{1}{\sqrt{I_{DS}}} \quad (\text{CS amplifier})$
- The high voltage gain is achieved by the large β_1 or small I_{DS} , but the small I_{DS} or small Δ_{OV} increases influence of the process variation and the large β_1 or large W causes a **Miller effect**.
- The cascode technique is useful for the gain enhancement by increasing the output resistance of amplifiers without additional bias current.

Miller effect

The AC characteristic of amplifiers is remarkably degraded by the Miller effect.



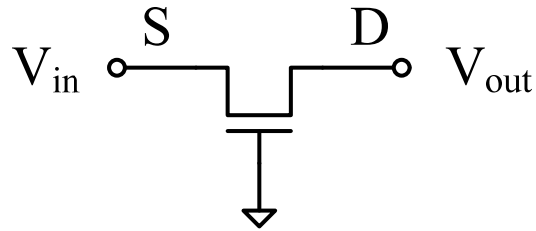
$$\begin{cases} v_{out} = -A \cdot v_{in} \\ i_{in} = j\omega \cdot C(v_{in} - v_{out}) = j\omega \cdot C \cdot (A+1) \cdot v_{in} \end{cases}$$

$$R_{in} = \frac{v_{in}}{i_{in}} = \frac{1}{j\omega \cdot C \cdot (A+1)}$$

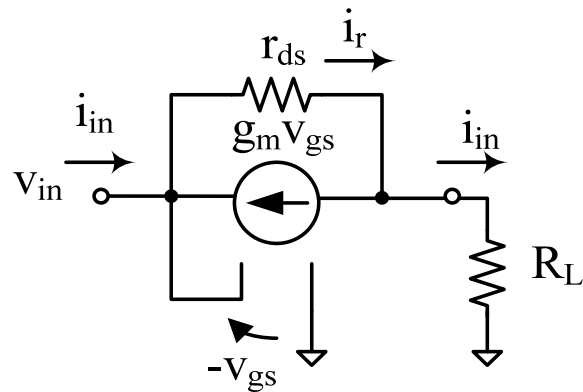
Equivalent circuit

Assuming that the input impedance is very large.

Trans-impedance with CG amplifier



Voltage gain of CG amplifier: $A_V = 1 + g_m \cdot r_{ds}$



Input Impedance

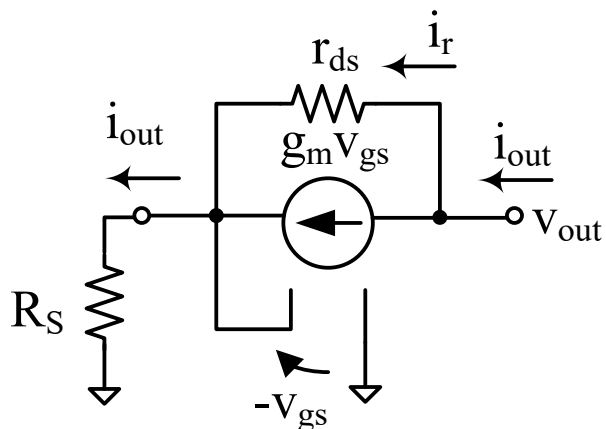
$$\begin{cases} v_{in} = -v_{gs} \\ i_{in} + g_m v_{gs} = i_r \\ v_{in} = r_{ds} i_r + R_L i \end{cases}$$

$$R_{in} = \frac{v_{in}}{i_{in}} = \frac{r_{ds} + R_L}{1 + g_m r_{ds}} = \frac{r_{ds} + R_L}{A_V} \approx \underbrace{\frac{1}{g_m}}_{\text{Memorize}} \quad (\text{if } R_L = 0)$$

Output Impedance

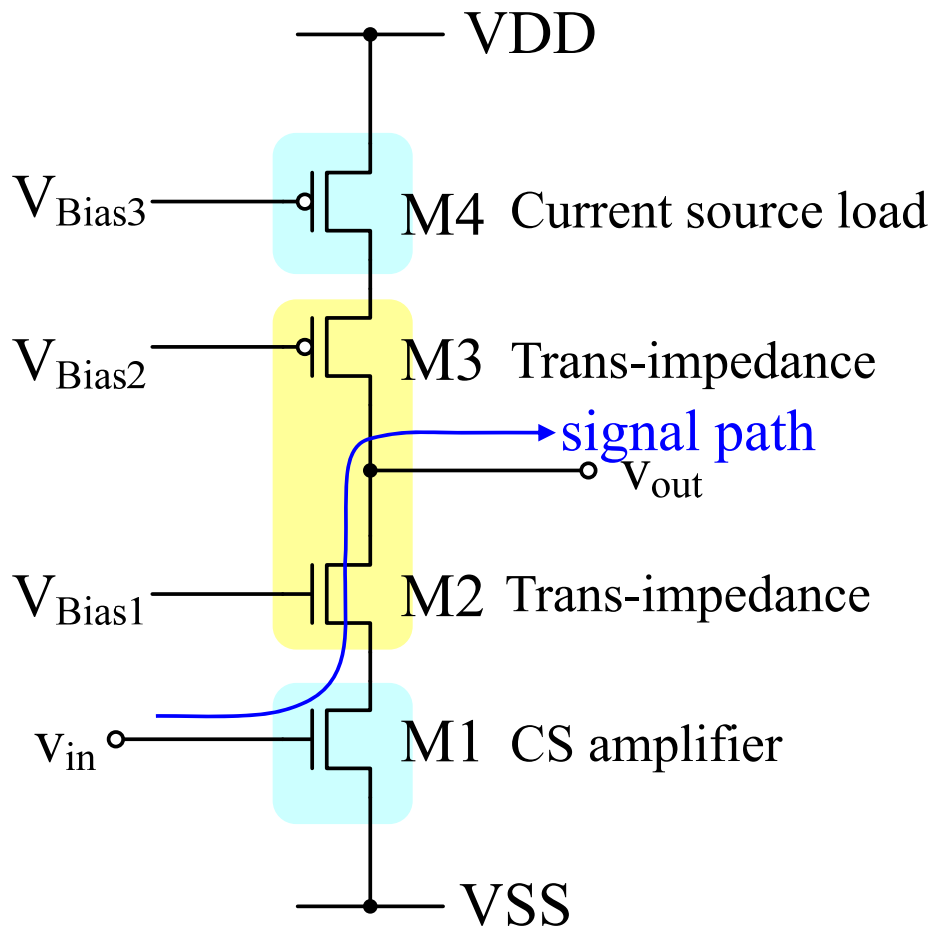
$$\begin{cases} -v_{gs} = R_S i_{out} \\ i_{out} = g_m v_{gs} + i_r \\ v_{out} = r_{ds} i_r + R_S i_{out} \end{cases}$$

$$R_{out} = \frac{v_{out}}{i_{out}} = r_{ds} (1 + g_m R_S) + R_S \approx \underbrace{A_V R_S}_{\text{Memorize}}$$



Memorize₁₉

Cascode amplifier



$$A_V = G_m R_{out}$$

$$G_m = -g_{m1}$$

$$R_{out} = \{(g_{m2}r_{ds2})r_{ds1}\} // \{(g_{m3}r_{ds3})r_{ds4}\}$$

If $r_{ds1} = r_{ds4}, r_{ds2} = r_{ds3}, g_{m2} = g_{m3},$

$$A_V = -\frac{1}{2} g_{m1} r_{ds1} g_{m2} r_{ds2}$$

Voltage gain of CS amp. $\sim 30\text{dB}$ ($=g_{m1} \cdot r_{ds1}$)

Voltage gain of cascode amp. $> 60\text{dB}$
without additional power consumption.

However, the cascode circuit designed for low VDD is disadvantageous for the output swing, because the stacked MOFETs should be driven in the saturation region.

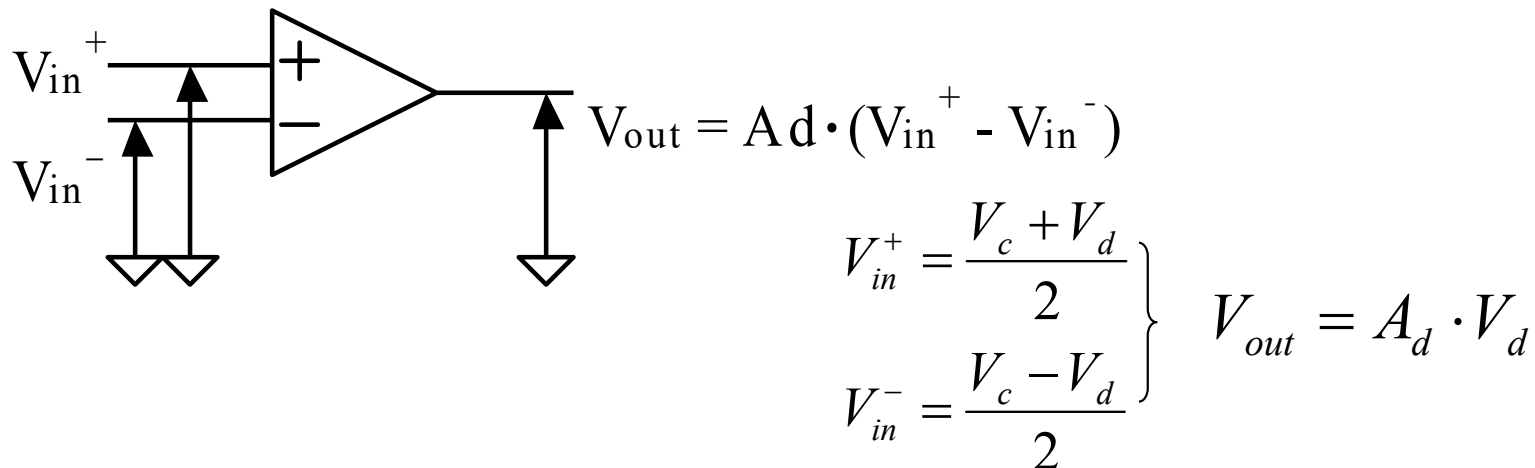
Advantages of cascode amplifier

- The bias current of CG amplifiers is reused for the bias current of the CS amplifier.
- The unity gain frequency of cascode amplifier is higher than other amplifiers.
 - The Miller effect of the CG amplifier is eliminated, because of the very small parasitic capacitance between S(input) and D(output). (See Chapter 11.)
 - The Miller effect of the CS amplifier is eliminated too, because the voltage gain of the CS amplifier is ~ 0 dB.
 - The voltage gain of the CS amplifier is reduced by the very low input impedance of the succeed CG amplifier.
 - The CS amplifier in the cascode amplifier is working as a trans-impedance amplifier.

10.3 Differential amplifier

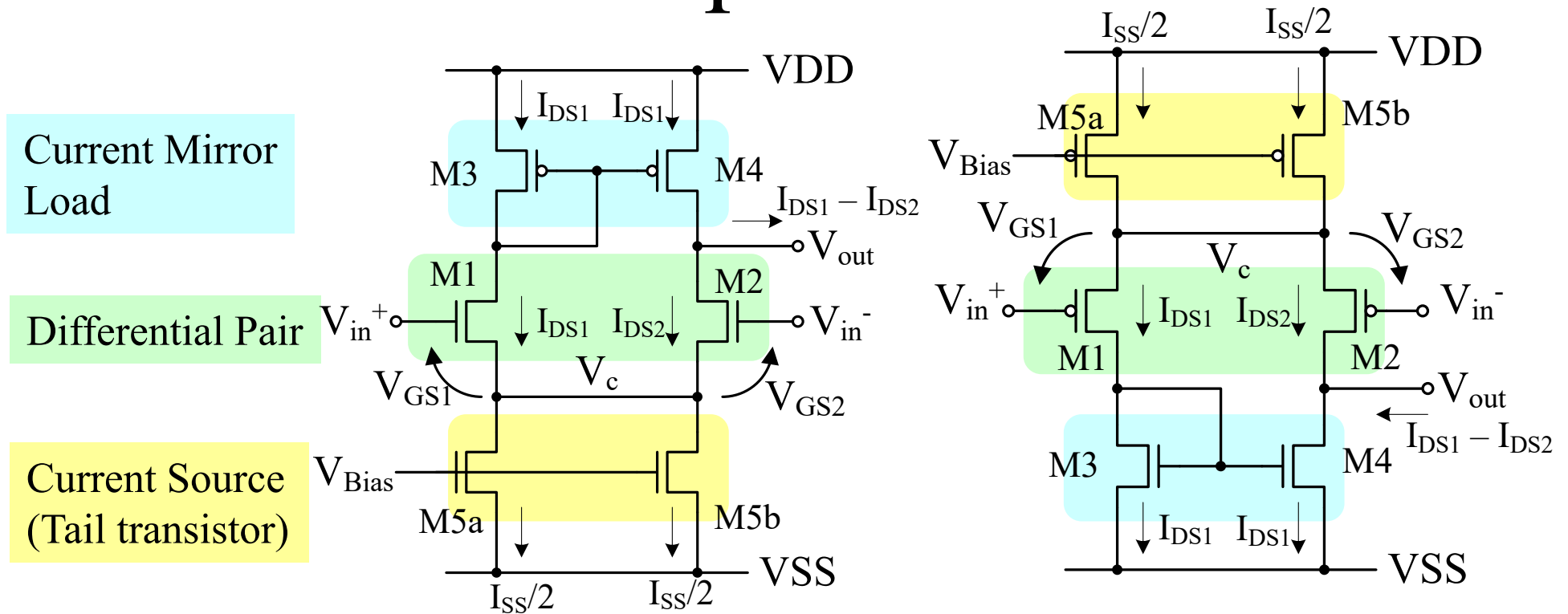
Function of differential amplifier

- The differential signal $V_d = (V_{in}^+ - V_{in}^-)/2$
 - is amplified to A_d times.
- The common-mode signal $V_c = (V_{in}^+ + V_{in}^-)/2$
 - is eliminated.



- Advantages of the differential amplifier
 - The common-mode errors included in a signal is eliminated.
 - Common-mode errors: Switching noise, Crosstalk, Thermal drift, Offset, Even-order distortion

Configuration of the differential amplifier



$$I_{DS1} = I_{DS2} = I_{SS}/2.$$

All MOSFETs are driven in the saturation region.

The noise level of p-ch MOSFET is smaller than that of n-ch MOSFET.

Differential gain

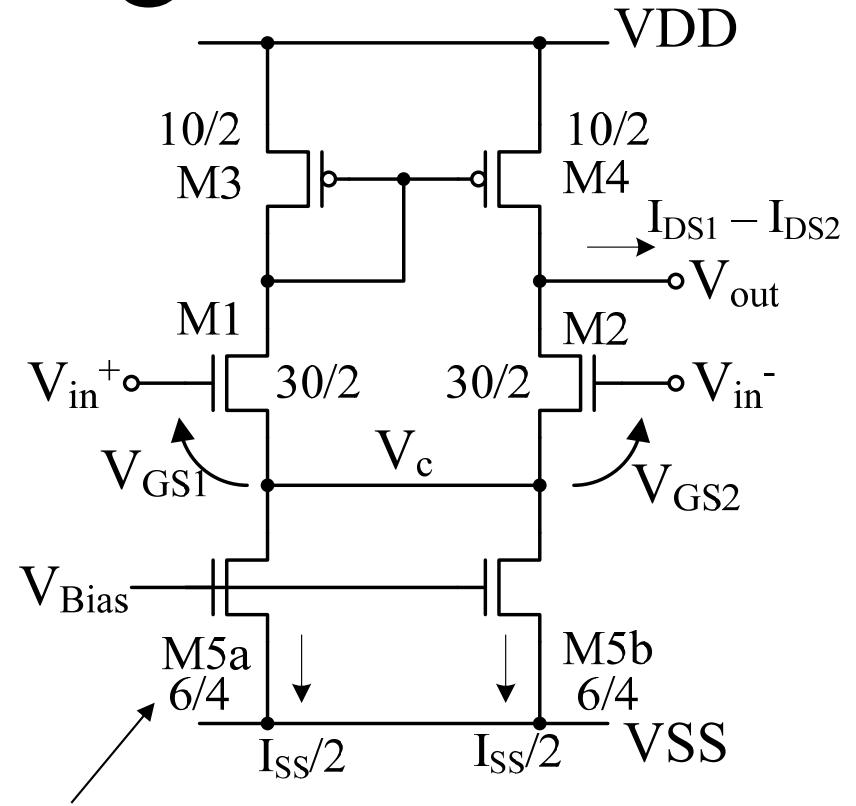
$$\begin{cases} i_{DS1} = g_{m1} (v_{in}^+ - v_C) \\ i_{DS2} = g_{m1} \cdot (v_{in}^- - v_C) \end{cases}$$

$$\begin{aligned} i_{out} &= i_{DS1} - i_{DS2} \\ &= g_{m1} \cdot (v_{in}^+ - v_{in}^-) \end{aligned}$$

$$G_m \equiv - \left. \frac{i_{out}}{v_{in}^+ - v_{in}^-} \right|_{v_{out}=0} = g_{m1}$$

$$R_{out} \equiv \left. \frac{v_{out}}{i_{out}} \right|_{v_{in}=0} = \frac{1}{g_{ds1} + g_{ds2}} = r_{ds2} // r_{ds4}$$

$$A_d = G_m \cdot R_{out} = g_{m1} (r_{ds2} // r_{ds4})$$



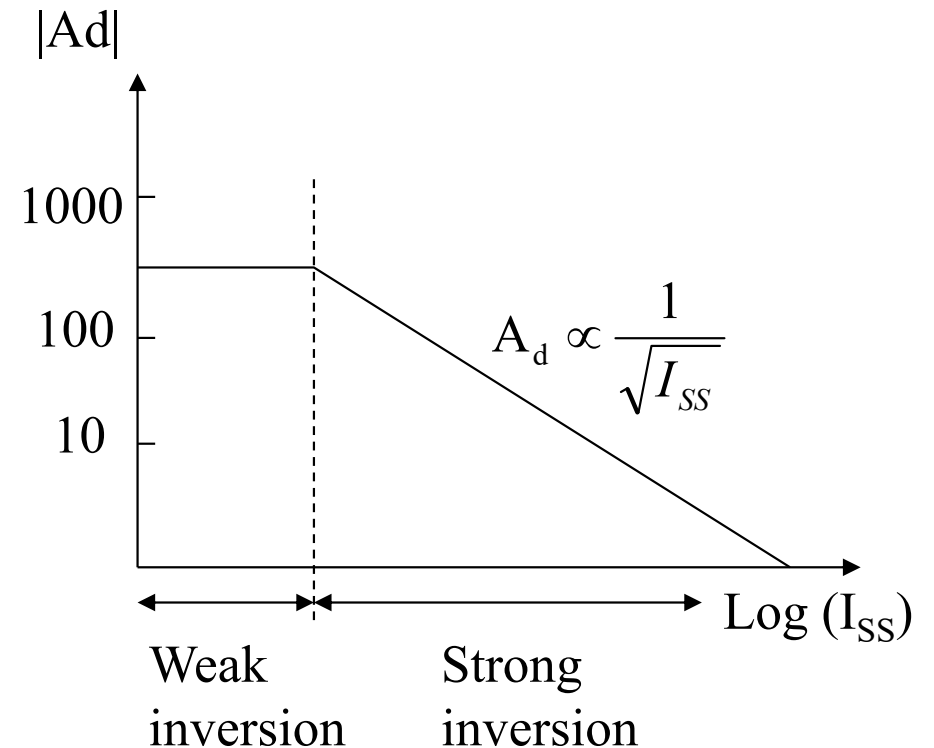
The large L of M5 can suppress the channel length modulation effect.

($V_C = \text{const.}$ The source node of M1 and M2 is equivalent to GND for the differential input signal.)

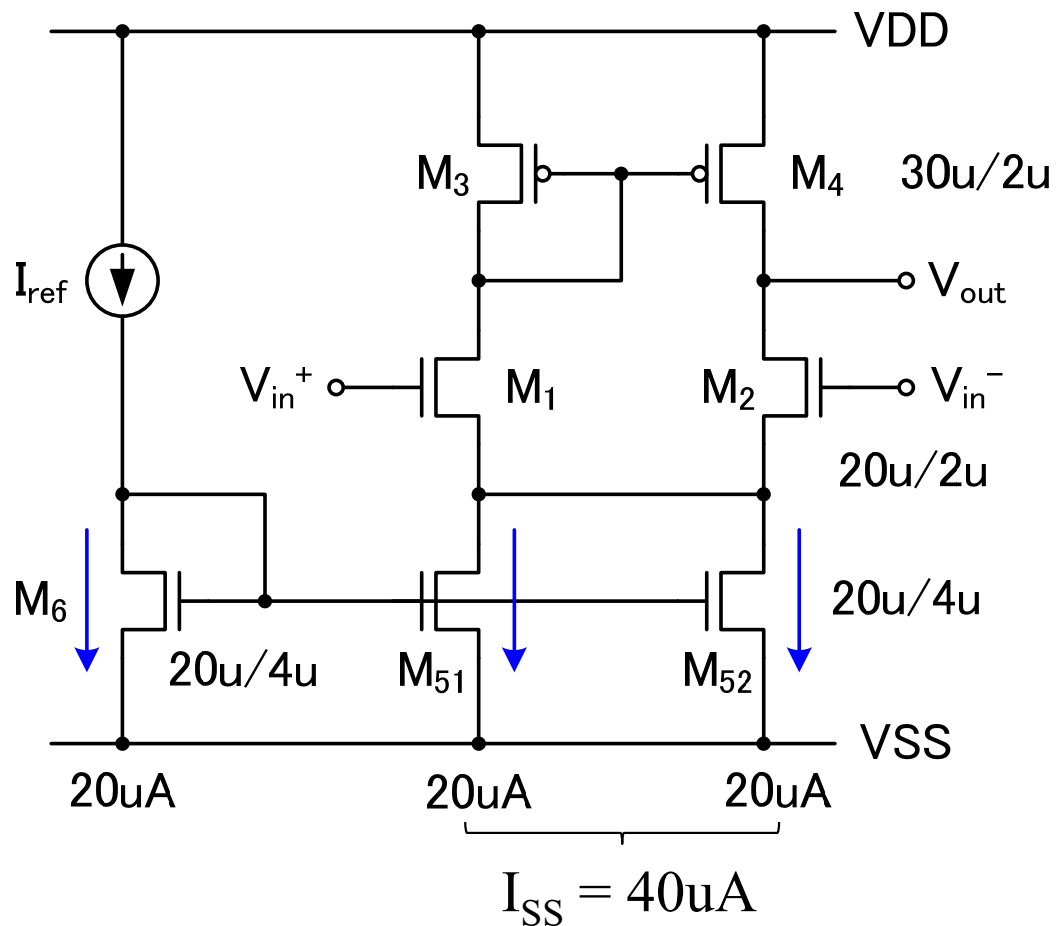
Setting of the bias current

$$\begin{aligned}
 A_d &= \sqrt{2\beta_1 I_{DS1}} \cdot \frac{1}{\lambda_2 + \lambda_4} \frac{1}{I_{DS1}} \\
 &= \frac{\sqrt{2\beta_1}}{\lambda_2 + \lambda_4} \sqrt{\frac{2}{I_{SS}}} \\
 &= \frac{2\sqrt{\beta_1}}{\lambda_2 + \lambda_4} \frac{1}{\sqrt{I_{SS}}}
 \end{aligned}$$

(without the external load)



Design example



$$\Delta_{OV1,2} = 0.2V$$

$$\lambda_1 = 0.019V^{-1}$$

$$\lambda_3 = 0.011V^{-1}$$

$$\beta_1 = 98\mu * 20\mu/2\mu$$

$$= 980\mu A/V^2$$

$$A_d = \frac{2\sqrt{\beta_1}}{\lambda_2 + \lambda_4} \frac{1}{\sqrt{I_{SS}}}$$

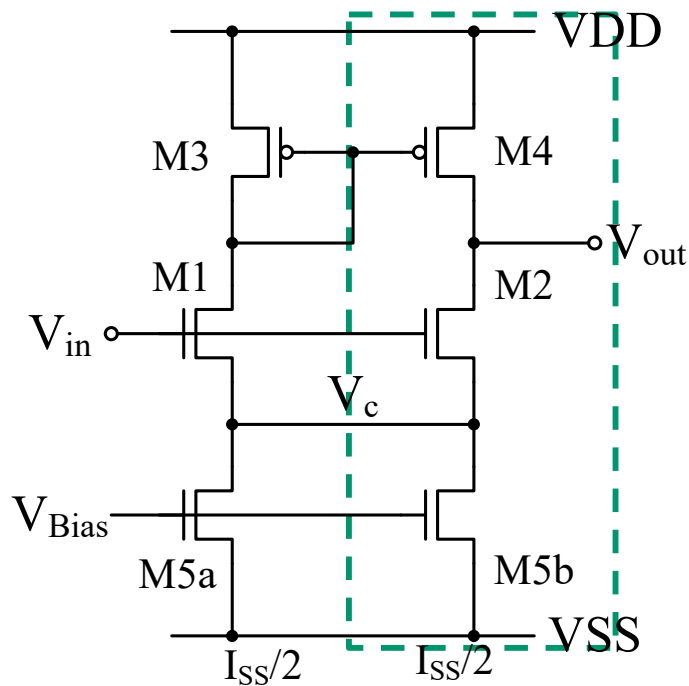
$$= \frac{2\sqrt{980 \cdot 10^{-6} A/V^2}}{0.019V^{-1} + 0.011V^{-1}} \frac{1}{\sqrt{40 \cdot 10^{-6} A}}$$

$$= 330$$

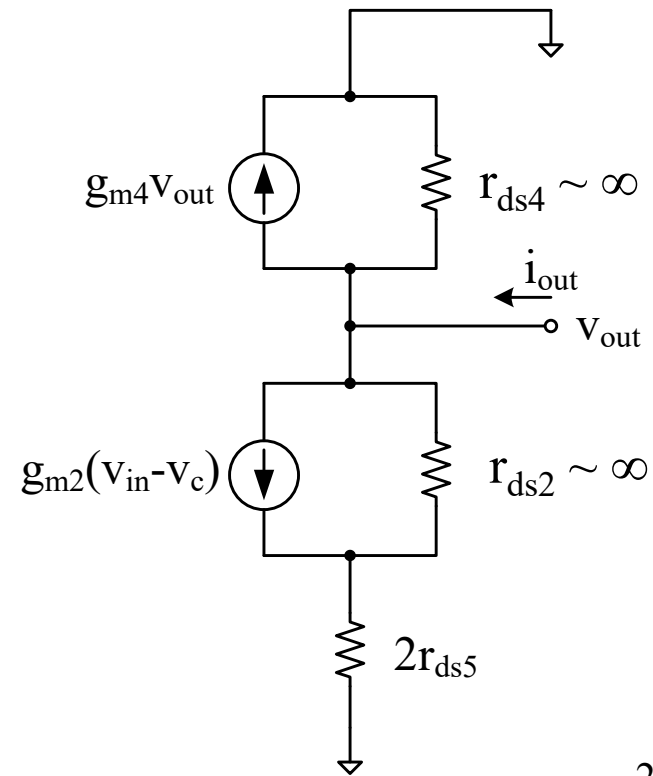
$$= 50.4dB$$

Cause of the common mode gain (Ideally $A_C = 0$)

1. The variation of V_C by the common-mode input.
2. The mismatch of the pair transistor M1 and M2, M3 and M4



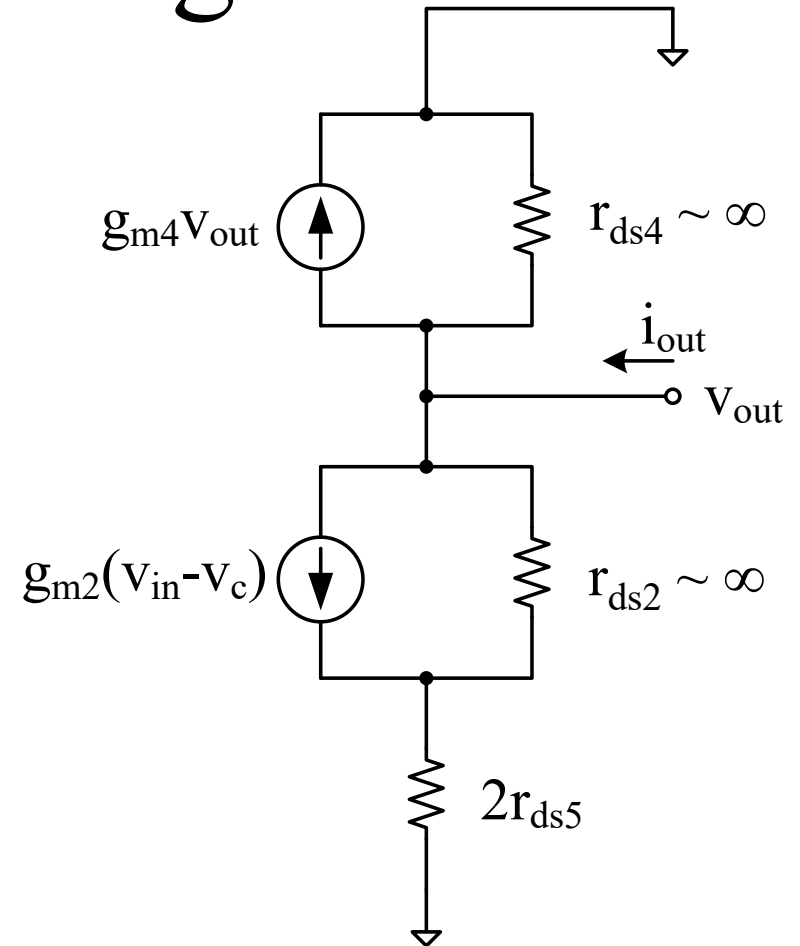
Small-signal



Common mode gain

$$\begin{cases} i_{out} = g_{m2} \cdot (v_{in} - v_C) + g_{m4} v_{out} \\ v_C = 2r_{ds5} g_{m2} (v_{in} - v_C) \\ i_{out} = 0 \quad \text{without external load} \end{cases}$$

$$\begin{aligned} A_c &\equiv \frac{v_{out}}{v_{in}} = -\frac{g_{m2}}{g_{m4}} \frac{1}{1 + 2g_{m2}r_{ds5}} \\ &= -\frac{g_{m1}}{g_{m3}} \frac{1}{1 + 2g_{m1}r_{ds5}} \end{aligned}$$



To reduce A_C , the large g_{m3} , g_{m4} and r_{ds5} is preferable.
The cascode current mirror is effective to increase r_{ds5} .

CMRR(Common mode rejection ratio)

$$CMRR \equiv \left| \frac{A_d}{A_c} \right|$$

$$v_{out} = A_d \cdot (v_{in}^+ - v_{in}^-) + \frac{1}{2} A_c \cdot (v_{in}^+ + v_{in}^-) + V_{OS_out}$$

$$= A_d \cdot (v_{in}^+ - v_{in}^-) + A_d \left\{ \frac{1}{2CMRR} \cdot (v_{in}^+ + v_{in}^-) + V_{OS_in} \right\}$$

Input-referred offset

common mode gain

process variation

The precise signal processing requires the larger CMRR.

Differential input range

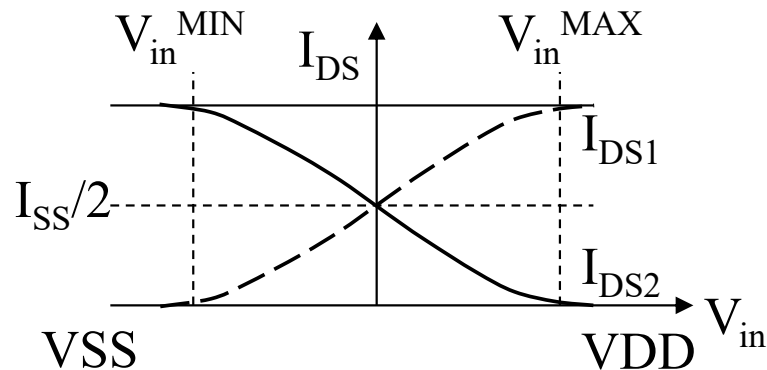
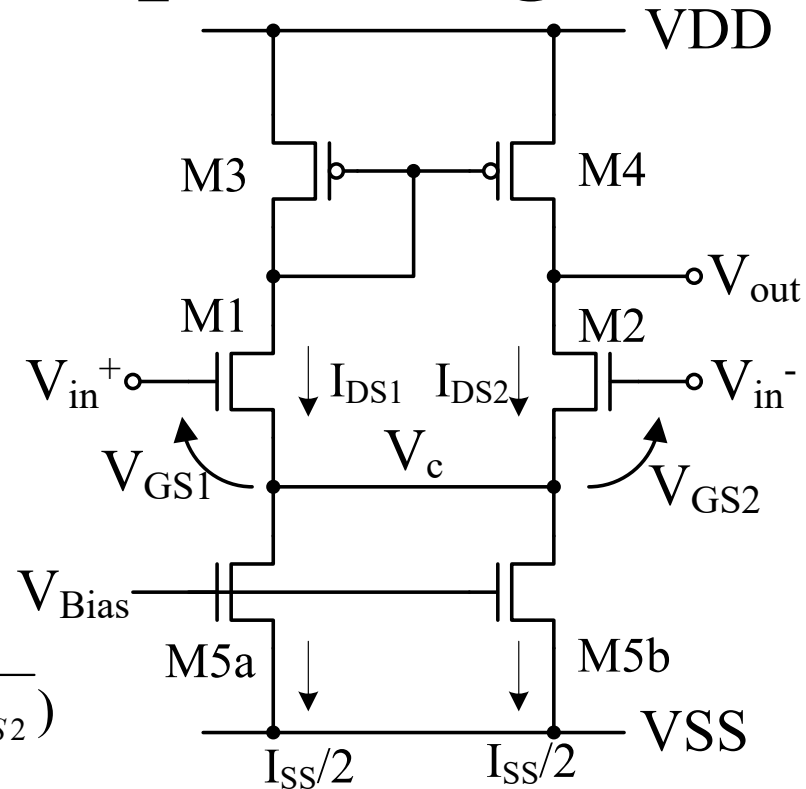
$$\begin{cases} V_{GS1} = V_{Tn} + \sqrt{\frac{2I_{DS1}}{\beta_1}} \\ V_{GS2} = V_{Tn} + \sqrt{\frac{2I_{DS2}}{\beta_1}} \end{cases}$$

$$I_{DS1} = I_{SS} \Rightarrow I_{DS2} = 0$$

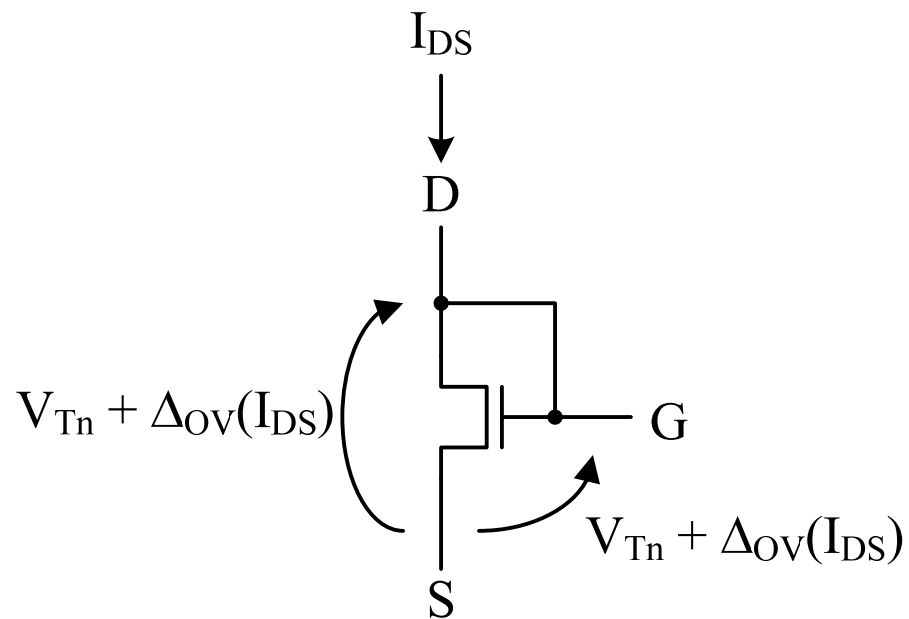
$$I_{DS2} = I_{SS} \Rightarrow I_{DS1} = 0$$

$$V_{in} = V_{GS1} - V_{GS2} = \sqrt{\frac{2}{\beta_1}} (\sqrt{I_{DS1}} - \sqrt{I_{DS2}})$$

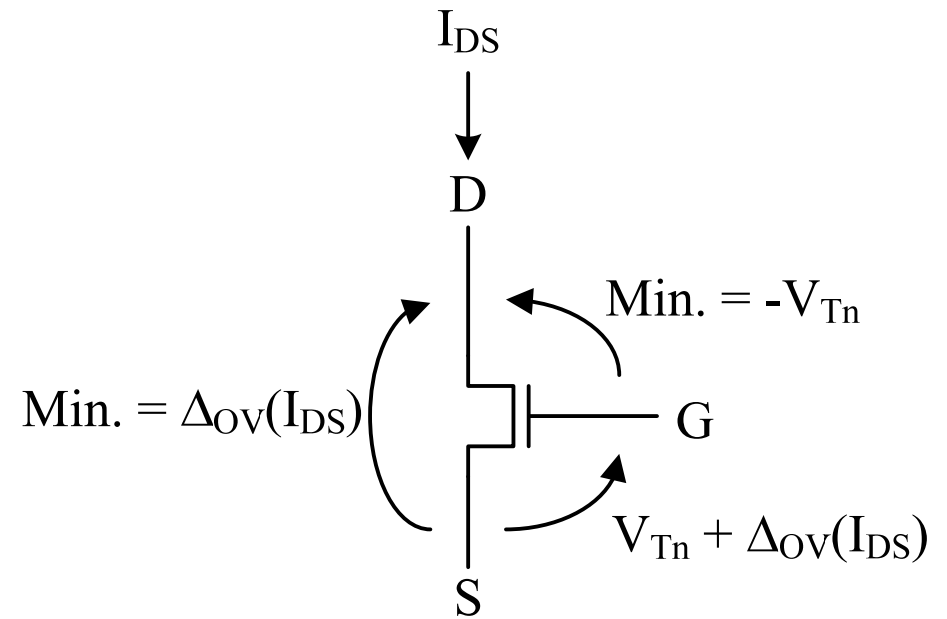
$$\begin{cases} V_{in}^{MAX} = \sqrt{\frac{2I_{SS}}{\beta_1}} \\ V_{in}^{MIN} = -\sqrt{\frac{2I_{SS}}{\beta_1}} \end{cases}$$



Calculation method of the common-mode input range



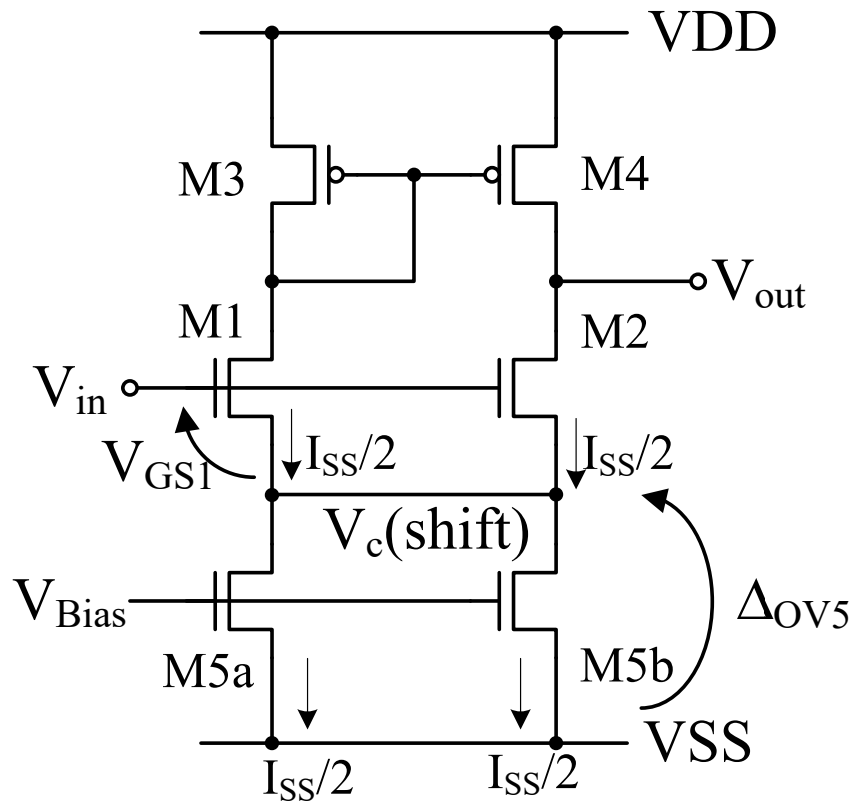
$$V_{DS} = V_{GS}$$



$$V_{DS} \neq V_{GS}$$

Common-mode input range (1)

Minimum common-mode input voltage



When both M1 and M2 is operating in the saturation region,

$$I_{DS1} = \frac{I_{SS}}{2} = \frac{\beta_1}{2} (V_{GS1} - V_{Tn})^2$$

$$V_{GS1} = V_{Tn} + \sqrt{\frac{I_{SS}}{\beta_1}} \quad (\text{Common-mode input})$$

While the M5 may operates in the linear region. if V_C is larger than $\Delta_{OV}(I_{SS})$ and M5 operates in the saturation region,

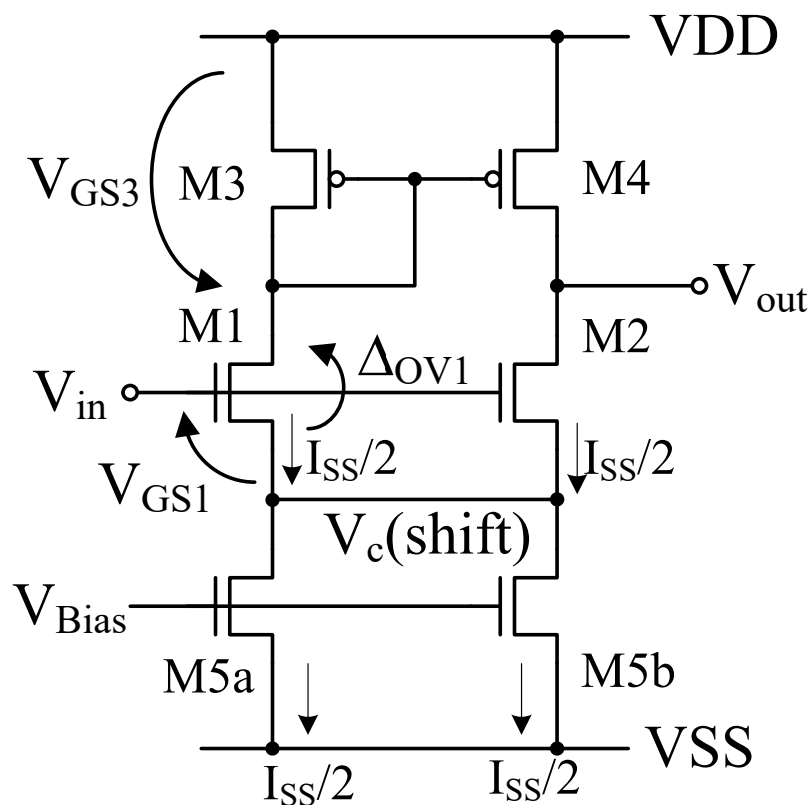
$$V_{DS5} \geq \Delta_{OV5} = \sqrt{\frac{2I_{SS}}{\beta_5}}$$

$$V_{in}^{\min} = V_{SS} + \Delta_{OV5} + V_{GS1}$$

$$= V_{SS} + V_{Tn} + \sqrt{\frac{I_{SS}}{\beta_1}} + \sqrt{\frac{2I_{SS}}{\beta_5}}$$

Common-mode input range (2)

Maximum common-mode input voltage



$V_{GS3} = V_{DS3}$, M3 and M4 is operating in the saturation region.

$$I_{DS3} = -\frac{I_{SS}}{2} = -\frac{\beta_3}{2} (V_{GS3} - V_{Tp})^2$$

$$V_{DS3} = V_{GS3} = V_{Tp} - \sqrt{\frac{I_{SS}}{\beta_3}} \quad (\text{Common-mode input})$$

Assuming that the M1 operates in the saturation region,

$$V_{DS1} \geq \Delta_{OV1} (I_{SS} / 2)$$

$$V_{in}^{\max} = VDD + V_{GS3} - \Delta_{OV1} + V_{GS1}$$

$$= VDD + V_{Tp} - \sqrt{\frac{I_{SS}}{\beta_3}} - \Delta_{OV1} + V_{Tn} + \Delta_{OV1}$$

$$= VDD + V_{Tp} + V_{Tn} - \sqrt{\frac{I_{SS}}{\beta_3}} \quad (V_{Tp} < 0)$$

Numerical calculation example of the common-mode input range

$$\underbrace{V_{SS} + \sqrt{\frac{2I_{SS}}{\beta_5}}}_{M5} + \underbrace{\sqrt{\frac{I_{SS}}{\beta_1}} + V_{Tn}}_{M1,M2} \leq V_{in} \leq V_{DD} - \underbrace{\sqrt{\frac{I_{SS}}{\beta_3}} - |V_{Tp}|}_{M3,M4} + \underbrace{V_{Tn}}_{M1,M2}$$

$$\left\{ \begin{array}{l} \Delta_{OV5} = \sqrt{\frac{2I_{SS}}{\beta_5}} \cong 0.3V \\ \Delta_{OV1,3} = \sqrt{\frac{I_{SS}}{\beta_{1,3}}} \cong 0.2V \\ |V_{Tp}| = V_{Tn} \cong 0.8V \end{array} \right.$$

